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# Mainstream Economics Under Austrian Assumptions: A Proposed Project

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# A Defense of the Project

Austrian economists have been fighting two long wars over micro-economics with the economics mainstream. They have been winning one of those wars and losing the other. The war Austrians have been winning is the war against an over reliance on empirical methods, the war they have been losing is the war over economic theory. In order to finally make headway in the war over theory Austrians must pursue new ways of interacting with mainstream micro that accept the importance of mathematical thinking and apply mathematical insights.

## War the First: Scientism

Scientism has been slowly but steadily receding for quite some time now. The move to try and ground macro-economics in micro theory[Kir81, p. 113] (spurred by Lucas more then by any Austrian[Bar]), the willingness to grant the reliance on economic intuition and even the pedagogy of basic economics (which has drifted towards explaining theory and away from simply stating laws and claiming they were inductively arrived at) is evidence that the economics profession has come a long way since the fall of logical positivism, scientism, rampant progressivism, and technocracy.

This is not to say that empirical economics has gone away. There are, notable exceptions to the general trend; the glamorous behavioural economics for example. However, for the most part, new empirical insights are not accepted without some sort of accepted theoretical mechanism. If no theoretical explanation can be found the new idea is ripped to shreds by a sceptical mainstream who will work to undermine the validity of the empirical work and to find theoretical explanations that often show the results do not mean what they initially seem to.

Whether they know it or not, most of the micro-economic mainstream theoreticians are claiming to come up with laws that are true a-priori. They make assumptions about human action and deduce from them general rules that are true contingent on the assumptions.

This is exactly what Austrians have advocated for, not a complete retreat from observations and inferences about the world. The historical/statistical and applied/institutionally contingent branches of economics rely on data, not just theory. However, data is useless without some framework to interpret it with. Mises saw the social sciences as primarily a handmaiden for history, the social sciences provide a true framework for interpreting data. As a result, even though data can inform new pieces of theory, elucidate what theory applies to a given situation and guide general assumptions about the magnitude of a given economic effect, it is still the case that theory is *logically prior* to history[Mis33]. This methodological outlook requires a generally stronger reliance on a theoretical framework to inform non-theoretical branches, and a generally less reliance on empirical economics to try to inform theory [LB06]. Austrians might want to move the mainstream further down this path, but they should admit that there has been progress.

With the mainstream moving away from scientism and towards deduction (from either stipulations or axioms) one might expect that Austrians would find more mainstream acceptance, but alas the opposite has happened. Even as the mainstream has slowly become more accepting of Austrian-esque methods, the Austrians have become more and more ostracised. Contra the old days of scientism this is not because Austrians are purely theoretical, rather it is because of differences between Austrians and mainstream theoretical economists (not empirical econometricians). This is not to discount macro-economic differences with the mainstream centring around Austrian business cycle theory or the over-

reliance on concepts like general equilibrium. Those macro-economic differences may be more important factors in the shunning of Austrian economists then micro-economic differences, however, they will not be dealt with in this paper.

## War the Second: Mathematical Methods

The differences between Austrians and the Mainstream Micro-economists that are more truly divisive can be divided into two categories. The first of these categorical differences is concerned with the form economic reasoning takes while the second is concerned with the substance of what the economists are saying. The difference over form comes from the intransigence of many Austrians to use mathematical symbols for abstraction.

Many Austrians, especially Rothbard, are skeptical of all mathematical methods, not just the ones that are used to make sense of empirical data [Rot60]. This is rather strange because Austrians hold that mathematics has much in common with economics. They are both deductive sciences. So the question becomes why not formalize deductions symbolically the way mathematicians do? Philosophy has an entire branch (symbolic logic) that does just this, and it exceedingly useful for finding holes in arguments as well as implications of ideas that otherwise would have been missed. If it's useful for philosophy, a discipline just as rooted in verbal deduction as economics, why would it not be useful for economics?

One reason is that the use of mathematics seems to many Austrians simply be part of the scientistic fallacy. It represents simply more "uncritical application of the method of the hard sciences to sciences dealing with man"[Rot60]. But this seems not to be the case. Mathematics pre-dates modern science and the empiricism of Bacon or Locke, and has more in common with economics and philosophy then with the empirical inductive sciences. But there is another

reason to be sceptical of the use of mathematics in economics.

Rothbard, in his Praxeology: The Methodology of Austrian Economics[Rot11], argues that the mathematical formulation of things should be rejected, not because it necessarily leads to false hood, but instead because it is unnecessary. he grants (and even argues) that mathematics and verbal deduction can express the same propositions and arguments, but since language is easily comprehended at every step it ought to be preferred. He appeals to Occams razor; why make a thing more complicated then it needs to be? Rothbard's mistake is in assuming that the symbols have no benefit for clarity, and that all they do is obfuscate, when in fact they can either obfuscate of clarify depending on their use.

It is true that mathematical formulations of economics can result in the economist taking steps that seem legal in the mathematical system but in fact make no sense when considering the meaning of the terms. However, it is also true that seeing the mathematical symbols can lead to perfectly valid manipulations and logical moves that would have been much harder to come up with working solely with words. It is true that for relatively short chains of reasoning mathematics may not contribute anything to the ease of understanding and may only obfuscate what is going on. But for longer chains of reasoning mathematics can be quite helpful for comprehension of what is going on. Mathematics is a tool that, with judicious use, could wonderfully extend economic science.

It should also be noted here that Rothbard's argument applies to other disciplines that use mathematics or symbols as well. everything said by mathematicians through symbolic mathematics could be said verbally, it would just take much longer to explain. Every thing philosophers say with symbolic logic could be said more understandably with words. Even the sciences of chemistry and physics could, if they were so inclined, say things more clearly with words. This might even result in a lower likelihood of making certain types of mistakes

(the way Rothbard suggests verbal deduction works in economics). For example, if you used words instead of symbols you would be less to divide by zero. This has never been much of a problem for these disciplines though, the benefits of abstraction through symbols have almost always outweigh it's cost. Over time rules are added to the system to make mistakes due to misapprehending the symbols meaning less likely. The same could be done with economics.

The project of the Austrians should not have been to advocate against the use of the tool of mathematics, but rather to advocate against its un-judicious use. Austrian theory and assumptions guiding the development of modern theoretical micro would have been quite a different animal from the micro theory found today. Mainstream economists reading Austrian writings perceive them as being fundamentally hostile to mathematical methods and thus they became unwilling to look at their implications for their mathematical procedure. The Austrian hostility is not entirely unwarranted: mainstream mathematical models do things they should not be able to do, and do obfuscate the meaning of their claims. Whether the focus on this aspect of things was prudent as matter of strategy is a different question entirely from whether it was true. The debate over form, and whether verbal logical deduction or mathematical notation is a better form is not a winning argument for Austrians and is not even particularly important.

The second category of differences deals with content rather than form. When the Austrians question the assumptions and techniques of the mainstream they are incredibly insightful and probing. The four main arguments against the mainstream mathematical tools (not against mathematics as a form of communication) are

- Whether utility should be treated in an ordinal or cardinal manner
- Whether goods should be treated as continuous or as marginal units

- Whether indifference should be used as an economic concept
- Whether the focus on equilibrium is warranted

The last item on the list is likely the most important, however it is not an issue that can be addressed at length in this paper. We should note though that the over reliance on equilibrium as an almost ethical good results in a radical shift of focus away from studying actual market processes, and undervalues the role of the entrepreneur by assuming that there is this fixed perfect out there that the market slowly moves towards[Kir81]. The mainstream developments in micro-foundations that we mentioned earlier, in tandem with the development of game theory as a branch of economics help to put some of the focus back on market processes.

The over focus on the nature of equilibrium is probably really at the core of the difference between the mainstream and most Austrians, and not the more dry and technical issues that the first three arguments focus on. However, the first three arguments are more fundamental, they must be answered before you ever arrive at the question of equilibrium. In a way they are more fatal flaws even, because if the Austrians are right there is no proper application of those ideas (while if Austrians are right about equilibrium it still has some roll in economic analysis). Equilibrium underlies most modern micro and macro economics, however, indifference curves, continuous units, and cardinal utility underlies virtually all of it. If the first three Austrian arguments are true an even larger swath of the edifice of mainstream micro might be undermined then whether the argument over equilibrium is true.

The reason the first three Austrian critiques have not been accepted by the mainstream is not because they are unconvincing. Rather, the reason they have not been widely accepted is because there are no easily arrived at mathematical alternatives to what is currently in use. Austrians present only the alternative

of returning to the use of verbal deduction as opposed to using mathematical symbols. There are good reasons for this, (the ones we talked about above), but if the mainstream will only take a symbolic formulation seriously why not just present them with one? Until Austrians do this the mainstream will labour under the delusion that mathematical economics cannot be done without the assumptions of indifference, cardinal utility, and infinitely divisible units of goods, and so they will continue to accept those assumptions.

If starting premises are true then anything that is validly deduced from them will also be true. This is true whether the deduction is verbal or symbolic. If Austrians could show how mainstream mathematical micro would work if the Austrian arguments against the assumptions are accepted the Austrian system may come to be seen as an actual alternative.

In doing mathematical economics we must be careful to meet two conditions the neo-classicals have failed to meet. First, the economist must make sure it is clear the conditions under which the deduced rules apply. Austrians are correct that this is something that mathematics can obfuscate. While the results of a model are almost always explained in plain English the implicit assumptions of the mathematics are not, which can be misleading even to the economist who did the work themselves. If assumptions are made as a matter of course then it is easy to forget they are being made.

Second, the economist must not set up impossible (or plainly false) conditions. The Austrians have critiqued many assumptions that are core to Neo-classical theoretical microeconomics as being either false or in some cases incoherent (The three listed earlier, though there may be others). While these differences about assumptions result in much strife between mainstream and Austrian economists, they can be confusing to those unfamiliar with the debate because Austrian and Neo-classical micro-economics yields many of the same

conclusions.

## The Project

In light of this, we believe a serious project should be pursued to reconstruct mathematical microeconomics along Austrian lines. The result of the undertaking would be the presentation of a clear alternative, the presentation of clear differences between the conclusions of neo-classical models and Austrian models, and the presentation of what it is we actually undertake when we undertake mathematical microeconomics.

This is obviously too large an undertaking for a single paper. So instead this paper will deal with the foundations of both views, their relations, and different paths of enquiry that might be pursued. No doubt many such paths are dead ends, but perhaps some will yield results. Even this though is too large an undertaking for a single paper, so in the end this is more likely to be a mere outline of what could be pursued in the future.

### Austrian and Mainstream Foundations

One of the primary difference between the foundation of the Austrian view and the Mainstream view is that the mainstream view starts with a model involving individuals choosing between bundles of different goods, while the Austrian view starts with a model where individuals choose between marginal units of one particular good and marginal units of a different particular good.

An Austrian might start for example with the following table

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $A$ |
| 4 | $2nd$ | Good $B$ |

Which is interpreted as meaning that the individual prefers the first unit of good $A$ to the first unit of good $B$, the first unit of good $B$ to the second unit of good $A$, and prefers the second unit of good $A$ to the second unit of good $B$, where being prefered is a transitive property[1] and an a-symmetrical property[2]. This ordering is not a necessary ordering, it could have been arranged a number of different ways. The person involved may have preferred both his first and second unit of good $B$ to all units of good $A$ for example. His ordering could have been inverted, going first unit of $B$, first unit of $A$, second unit of $B$, second unit of $A$. etc. A unit $x$ is said to be ranked more highly than a unit $y$ if and only if the ranking it is assigned (called $r_x$) is less then the ranking $y$ is assigned (called $r_y$) i.e. $r_x$ is closer to 1.

One main benefit of the Austrian system is that it strongly emphasizes diminishing marginal utility. Each additional marginal unit of good $A$ will always be ranked lower then the units already listed. If you have an apple it will go to the highest end an apple could fill for you, a second apple would go to a less highly valued end in every case. This law of economics is built into every possible Austrian style ranking. A rule for placing a unit of some good $x$ on the ranking is that if it is the $n$th unit of $x$ it must be ranked lower then the 1st through $(n-1)$th unit of $x$.

[1] if the $n$th unit of good $A$ is preferred to the $m$th unit of good $B$ and the $m$th unit of $B$ is preferred to the $l$th unit of $C$ then the $n$th unit of good $A$ is preferred to the $l$th unit of $C$

[2] if the $n$th unit of good $A$ is preferred to the $m$th unit of good $B$ then the $m$th unit of good $B$ is not preferred to the $n$th unit of good $A$

A mainstream economist on the other hand might start with something like this:

| $U_{AB}$ | $Q_A$ | $Q_B$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 0 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

Where $U_{AB}$ represents the ranking, $Q_A$ represents the quantity of some good $x$ and $Q_B$ represents the quantity of some good $y$

In this model, instead of ranking units of a good against one another we rank bundles of goods against one another, where a bundle is an $n$-tuple of the form $(x_1, x_2, ...x_n)$, where $x_1$ indicates $x_1$ units of good $A$, $x_2$ indicates $x_2$ units of good $B$, etc. In this paper we will only be dealing with 2-tuples and bundles of size 2. In other words we are only looking at 2 goods. In this model "being preferred" is also a transitive, a-symmetrical property. Another similarity to the Austrian model is that we stipulate which bundles are ranked more highly, or are preferred. A bundle $(x, y)$ is preffered to $(w, z)$ when the ranking of $(x, y)$, $r_x$ is less then the ranking of $(w, z)$, $r_w$. In the above example the person would prefer having one unit of good $B$ to having nothing, one unit of good $A$ to having one unit of good $B$, and having one of each to any of the alternatives. This also could have been arranged differently, so that the person preferred having one unit of good $B$ to one unit of good $A$ as so:

| **Ranking** | **Units of Good A** | **Units of Good B** |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 0 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

Just as the Austrian model requires that the $nth$ unit of a good be ranked lower then the 1st through $n-1$th unit of the good so the mainstream model requires that any bundle $(x, y)$ be ranked lower then any bundle $(x+n, y+m)$, where either $n > 0$ or $m > 0$ and both $n \geq 0$ and $m \geq 0$. Alternatively $(x, y)$ is always ranked higher than $(x-n, w)$ and $(z, y-m)$ where $w \geq y$ and $z \geq x$ and $n, m > 0$

These two models have more similarities then they have differences. They both have a relation of preference that is a-symmetrical and transitive, they both are arranged so that a ranking closer to one is a higher ranking, neither requires any particular ordering be a necessary ordering. Every real person has some ordering in both systems, since if any person is given a choice between two bundles of different goods, or a choice between units of any two of the goods they must make a choice. Both systems have in common that they are incomplete preference rankings, any person in the real world possesses an infinitely long list of this form, since there is some truth about but what they would choose if they were presented with a choice between two things not on our ranking (Preference is demonstrated in action).

While we can say with appodictive certainty that any person, in the actual world, when faced with a choice between one banana and one apple would have to choose either to accept the apple, or the banana or to refuse them both, we cannot say with any certainty which they would pick and so our incomplete orderings are a stipulation about what the ordering is. As a result examples should be recognized as examples. We need to be very careful about generalizing results from one or two examples to the entire set of all possible orderings. In general proofs about either ordering system should use the most general form possible, even though we recognize that specific examples may allow us to see things we would not notice otherwise about the entire set. This is not to say

that there are not some cases where we may stipulate something about some set of orderings without loss of generality. This is also not to say that in some cases we may want to stipulate certain things about the orderings we are dealing with and prove things about that particular family of orderings.

In the Austrian version units of a good are labled, first, second and so on in order to demonstrate the concept of diminishing marginal utility. The first apple you receive will be used to fulfill your highest valued end with that apple, any additional apples (even if they are used for the exact same thing) satisfy less highly valued ends then the first apple does[Baw60]. The mainstream system of ranking does not emphasize this fact, nor does it have any way of expressing it. The first, second, third, etc. units of any good are all represented in multiple bundles instead of only once, as a result there can be no distinction directly drawn from the ranking about the relative values of additional units of some good. Instead the mainstream ordering opts to focus on the economic law that more of a particular good is always preferred to less (the point of the mainstream rule above that determines what rankings are valid and which are invalid). The Austrian model also implies that more is always preferred to less, a ranking like the one above where the person possesses one of good $A$ is less desirable then one where the person possesses two of good $A$ or where they possess one of good $A$ and one of good $B$.

One common addition to the Austrian model is adding parenthesis to our notation to notate that the person does not at this time have the good in parenthesis

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $(A)$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $(A)$ |
| 4 | $2nd$ | Good $B$ |

This ranking expresses the information that the person in question possesses two units of good $B$ and no units of good $A$. This is a case where the person would willingly trade one unit of good $B$ for a unit of good $A$, in the case this trade was offered the persons ranking would then look like this:

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $(A)$ |
| 4 | $2nd$ | Good $(B)$ |

which is clearly a more desirable state of affairs for the person. Please note that even though it is not emphasized in the Austrian model more is still always preferred to less. If the person above had the option to choose between trading his second unit of $A$ away for one unit of $B$ or to someone else for two units of $B$ the second option would have resulted in a better final state of his preference ranking.

However, the Austrian model does admit of ambiguity. It is unclear whether the person would be willing to trade away two units of good $B$ for one unit of good $A$. Whether or not such trades would occur is indeterminate in the system and ad hoc rules need to be introduced to resolve the ambiguity and make the ranking as complete as the mainstream ranking which contains information about whether all such trades would be amenable to the person.

| Ranking | Number | Good |
|---|---|---|
| 1 | $1st$ | Good $(A)$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $(A)$ |
| 4 | $2nd$ | Good $B$ |

| Ranking | Number | Good |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $1st$ | Good $(B)$ |
| 3 | $2nd$ | Good $(A)$ |
| 4 | $2nd$ | Good $(B)$ |

it is unclear which final preference ranking is 'better'

Whether this is a virtue or flaw of the Austrian system is up for debate. While the ranking is less complete, the Austrian model stipulates fewer things about the specifics of the persons ranking, in order to deduce much of the same content. Stipulating less seems to be a large advantage for making more universal claims.

Another notable difference between the Austrian and Mainstream view is that in the Austrian Model there is an upper bound. Some good exists such that there is no good preferred to it. In the Austrian model there is no lower bound, you can always add an additional unit of a good and it will be ranked less highly then the previous unit of that good. In the Mainstream model the opposite is true (assuming that there are no negative values allowed), there is a lower bound, some bundle that you would prefer every other bundle too. You can always add an additional bundle though, which possesses one more unit of a particular good and is thus ranked more highly then another bundle.

Having outlined some of the similarities and differences between the two systems we can go into the differences that Austrian economists have singled out as meaningful and deserving of critique.

## Critique the First: Ordinal and Cardinal

The systems you were introduced to above both seem as though they are purely ordinal relations between items. In the Austrian ranking the first unit of $A$ was preferred to the first unit of $B$ and the first unit of $B$ was preferred to the

second unit of $A$ but one of those preferences was not any 'larger' than another. According to the Austrian school the only information the preference is meant to convey is that given the option any lower ranked item would be traded for a higher ranked item. Preferences make no claim on magnitude or distance. They may in some psychological sense but not in an economic sense, in the economic sense all we can say is that $x$ is preferred $y$ and not that $x$ is preferred to $y$ two times more than $y$ is preferred to $z$.

The mainstream ranking could be similarly interpreted, but it has not always been interpreted so. The mainstream rankings tend to be generated by utility functions. functions that take a bundle $(x, y)$ and return a single cardinal value, say $k = U(x, y)$ is our utility function. That function is used to represent the entire ranking if $k_{1,0} > k_{0,1}$ then the bundle $(1, 0) > (0, 1)$ in the ranking. Many economists throughout history have used these functions as doing more then merely abstractly representing an ordinal ranking, instead they took the value of $k$ to represent some real psychological quantity and would say things like "Bundle $A$ is 3 times as valuable to person $Q$ as bundle $B$, however to person $P$ bundle $B$ is 2 times as valuable as bundle $A$ thus if they were to trade $Q$ would benefit more than $P$" Such claims are absurd and make no economic sense. You cannot compare utilities this way either inter personally or intra personally.

However, this is not how most modern mainstream economists interpret utility rankings. This is one theoretical battle Austrians have mostly won. Though economists still use utility functions to represent an entire ranking they try not to think of them as representing any sort of real distance of value between bundles, although they may sometimes lapse into that mode of thought.

Certain commonly held mainstream results rely on cardinal utility to hold. The Neuman-Morganstern models that attempt to explain choice under uncertainty rely on assuming cardinal utilities, although some complex mental

gymnastics has been done to argue that this is not necessary, although there does not seem to be consensus. Additionally, almost all of mainstream wellfare analysis relies on not just cardinal utilities, but interpersonal comparison of those utilities[Rot56]. Other mainstream attempts have been made to save the assumption or something close to it. Fisher argued that if the 'psychic satisfaction' gained from a good is independent of the consumption of other goods most of the things that can be done if utility is cardinal can still be done[Man99]. This would seem to most Austrians to be an even more plainly false assumption than cardinal utility since every good is a weak substitute or compliment for every other good, so we should certainly not expect the satisfaction from a good to be independent of the consumption of other goods. In order to get fisher's results you also have to make other poor assumptions, like that peoples utility functions remain constant over time.

Instead of using functions to construct Austrian rankings, Austrians have always stipulated their rankings directly. This again goes to the issue earlier of being more general and stipulating as little as possible so that the results apply as generally as possible. When you have a utility function you have a much more complete ranking, an infinite number of bundles are stipulated to be related to each other in a particular way for the actor. This Potentially lose more of the general applicability of a given example, then only stipulating a small relevant portion of the ranking of bundles.

## Critique the Second: Continuous and Marginal

The second critique Austrians generally level against mainstream economists is that mainstream economists generally work with infinitely divisible goods, whereas Austrians suggest that the legal values for the quantities of goods should be limited to $\mathbb{Z}^+$. The Austrian reasoning for this is that people decide between

heterogeneous goods not infinitely divisible goods. You will never buy $\frac{1}{100}$th of a pen, because the relevant unit for action is one pen. Similarly a supermarket purchasing pens does not purchase individual pens, because in this context a single pen is not a relevant unit of action for the supermarket, rather pens are purchased by the hundreds or some other large number. Austrians want to limit the numbers used to the integers so that the single unit can represent the economic unit relevant for action.

Mainstream mathematical micro-ignores this critique so that the utility functions they use can be continuous. In that way they can use calculus to manipulate the functions, finding things like there slope, convexity or concavity etc. Losing this would seem like a major loss to them because so much of mainstream theoretical microeconomics is built on this ability. This is one of the major reasons a project like this is needed. Modern mathematics has come a long way in dealing with discrete non-continuous entities, and Austrians, who for so long have insisted that economics should use discrete entities, should be at the forefront of developing a microeconomic system that does just that.

## Critique the Third: Indifference

The traditional mainstream view allows multiple bundles to share a particular ranking, it not only allows this, it uses this to do many different things. The state of two different bundles sharing a ranking is called indifference.

| $U_{xy}$ | $Q_x$ | $Q_y$ | $Q_x$ | $Q_y$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 0 | 0 |

Mainstream micro-economists use this concept to build indifference curves which have many applications from building demand curves to calculating in-

come and substitution effects. However, the concept of indifference is incoherent and has no meaning.

To say an actor is indifferent between $A$ and $B$ means that if he were presented with $A$ and $B$ he would choose... what? If he picked $A$ that would demonstrate that in fact $A$ was preferred to $B$ by the definition of preference. If he chose $B$ that would demonstrate that $B$ was preferred to $A$. All preference means is that when presented with a choice between two things the preferred thing will be chosen, so if a person is presented a choice between $A$ and $B$, he must have a preference, thus the concept of indifference is vacuous and meaningless. This is why Austrians insist that all preference rankings are complete. There is some truth of the matter of what every person would choose when given a choice between two things, and so every preference ranking is complete. This throws a major monkey wrench in theoretical micro-economics since so much of it is built upon this meaningless concept.

The idea of indifference may be necessary to keep around in some guise so that we can compare a good with itself. Recall that preference is transitive and a-symmetric, but we said nothing about whether it was reflexive. Suppose it was reflexive and you prefer bundle $A$ to itself, bundle $A$, not a second identical bundle, but literally itself, this would mean that our ranking would end up becoming nothing but an infinite repeating list of bundle $A$ after bundle $A$. However, if it is not reflexive and bundle A is not preferred to itself then there exists two bundles, $A$ and $A$, such that they have no relationship to each other violating the rule that all rankings are in reality complete. However, it is unclear what this actually means. does saying that a person is indifferent between choosing mint ice cream and the same unit of mint ice-cream actually mean that there has been actual indifference in some sense? Or does it simply mean that since indifference is an impossible state of affairs, it is similarly an impossible

state of affairs for a person to be forced to choose between a thing and itself? I'm inclined to hold to the second one. All the Austrian critique states is that if a person is ever given a choice between mutually exclusive courses of action they will choose one and thus demonstrate their preference for that action

If we were to reject the possibility of indifference as the Austrian suggests we are left with fewer acceptable rankings of bundles. Additionally it can be difficult to formulate a utility function that never yields indifference

## Other Critiques

There are other Austrian critiques of mainstream theoretical microeconomics, but these are the three primary ones, however, since rankings of bundles play such a major role in this project their validity needs to be addressed. Peter Klein has argued against them on the grounds that each bundle actually represents a completely different good and that people in action do not choose between bundles but between marginal units of two different goods that they are trading off between[Kle09].

It is true that each bundle represents a completely different good, but as established earlier, and as Austrians have argued for some time rankings must be complete. Which means that, even though two bundles are different goods, if in action you were choosing between two bundles you would have to make a choice. Ranking them is not incoherent, any person demonstrates a ranking of them in action. Additionally, unlike most things that are completely different goods, there is a necessary structure to a persons preferences about the bundles. The bundle $(x, y)$ will always be ranked lower then the bundle $(x + 1, y)$ or the bundle $(x, y + 1)$. Because every bundle must be ranked either higher or lower than another (because we ruled out indifference) it seems as those stipulating a particular ranking is just as valid as when Austrians stipulate a particular

ranking.

Klein's second critique that people in action do not choose between bundles but between marginal units of two different goods that they are trading off between may be a stronger argument. It is unclear whether it is a more psychological claim in nature or a more praxeological one. It seems as though whether or not people in the course of their action are evaluating between trade-offs between individual units of two things or between baskets of goods that contain a certain number of one good and a different number of another, comes down to how they think. It seems as though sometimes they think "do I want two of this and one of that, or do I want one of this and three of that?" it seems as though other times they think in terms of trading off units of less value for units of more value. This would be a psychological issue unrelated to praxeology and pure theoretical economics.

However, the argument could be taken in a praxeological way, in which case it might carry weight, but some skepticism is warranted. It seems as though in action people do both things simultaneously. They, every time they buy something, move from one market basket to another preferred market basket. They also, in an equivalent way, rearrange which things they possess on their Austrian scale so that it is in a better state. It seems as though the two things are the same thing and not mutually exclusive. Unlike the other critiques here I don't think it changes much, the only question is which way of arranging the way we think about action is more fruitful. Both styles of ranking seem equally 'like action' and are equally praxeologically valid.

Besides Kleins critique we must address the issue of bringing money into things. Unless we state otherwise we will assume that the only thing we know is the ranking given. Generally Austrians only have two goods in a preference ranking they are showing at once, and quite often one of those goods is money

and the Austrians use the ranking to deduce how prices are set. It is not necessarily the case that only two goods are in the ranking at a time, Bohm Bawerk stipulates more then two ends being ranked against one another and being fulfilled by multiple things[Baw60]. Additionally in action a choice reveals a preference for what is chosen over all the opportunity costs (although reveals nothing about the relative rankings of the opportunity costs to one another). There are some cases though were we will use money and in these cases we can either include the units of money in the ranking or imply things by stipulation about where money is ranked.

Neo-Classical economists generally take prices as a given for their actor(this seems valid since in the real world of action this is what people are most often faced with), and then give their actor a set budget constraint. The budget constraint can be thought of as an amount of money that the actor is stipulated to universally desire less then the goods he can buy for this part of the range of his preferences. The budget constraint is a pretty large stipulation that is being made, it is essentially stipulating that the ranking of bundles of two goods and money looks like this:

| $U_{xym}$ | $Q_x$ | $Q_y$ | $Q_m$ |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 2 |
| 3 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 0 | 1 | 2 |
| 6 | 0 | 1 | 1 |
| 7 | 0 | 1 | 0 |
| 8 | 0 | 0 | 2 |
| 9 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 |

Which seems like a pretty large stipulation. Additionally, it may be a useful tool to use a budget constraint method like this on the Austrian ranking system solely for the purposes of comparing the two systems in the future. A similar example for the Austrian would be

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $A$ |
| 4 | $2nd$ | Good $B$ |
| 5 | $1st$ | Money |
| 6 | $2nd$ | Money |

## Outline of the Project

In the quest to create a mathematical economic system using mainstream tools but without using the false assumptions critiqued by the Austrians there are no clear first steps. However, the first thing that we might want to do is find out some basic properties of the thing we're dealing with. The mainstream was dealing with incomplete rankings where multiple bundles were occupying the same level, now we're occupying a much smaller world with fewer options for where to place bundles. One way to explore this world would be to see how large it is, how many options there are for valid rankings of bundles of goods without indifference. If we can get our head around that we may be able to begin to divide these rankings of bundles into some that have some properties and others that have different ones. We may be able to compare the world of bundle rankings to the world of the Austrian ranking system and see if there is some easy way to draw a correspondence between the two. Perhaps each Austrian ranking corresponds to one or more modified neo-classical ranking, perhaps

Austrian rankings correspond to only some modified neo-classical rankings and others are not allowed under the Austrian strictures. These are all reasons why we decided that trying to count the number of possible modified neo-classical rankings was a good way to start.

## Counting the Number of Possible Rankings

Remember that the entire complete ranking is infinitely long, so counting the ways to build one of them makes no sense. Instead, what we are trying to count is the number of ways to count things over a mere portion of the complete ranking.

There are two possible ways we could go about this. One way would be to ask the number of ways that you are allowed to rank a certain set of bundles. Another way would be to ask how many different ways would there be to create a ranking over $x$ many places without having a place where you are required to put another bundle. For the purpose of this paper we are only dealing with cases of two goods but what we find would be important to generalize to cases with more 3 or more goods.

The first way we could go about trying to count is by possible orderings of a certain number of bundles, where it's possible to have them all ordered without anything else being forced to go between two of them. The way to generate these lists of bundles that we are going order on our preference rank would be to create all the bundles $(x, y)$ where $(x \leq X)$ for some $X$ and $(y \leq Y)$ for some $Y$. There are $(X + 1)(Y + 1)$ different bundles to rank in these cases. Here is the example for $(X = 2, Y = 2)$

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 1 | 1 |
| 6 | 1 | 0 |
| 7 | 0 | 2 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

Our goal is to count the ways you could rank these bundles such that if $(x_1, y_1) < (x_2, y_2)$ then it is **not** the case that $x_1 \geq x_2$ and **not** the case that $y_1 \geq y_2$

This seems to be a linear extension problem, and so prohibitively difficult to come up with a formula with through basic combinatorial methods. So instead we wrote a computer program to by brute force count the number of ways to order these acceptably. They seem to follow the following pattern $C_{x+1}(y+1)$ counts the number of possible complete bundle rankings. $C_d(n)$ represents the $n$th term of a $d$ dimensional Catalan number. the 2 dimensional Catalan numbers are given by $\binom{2n}{n}(\frac{1}{n+1})$ the 3 dimensional Catalan numbers are given by $\binom{3n}{n}(\frac{1}{2n+1})$ the d dimensional Catalan numbers are given by $\binom{dn}{n}(\frac{1}{(d-1)n+1})$

Values computed by our program are in **bold**, predicted values are *italicized*

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | **2** | **5** | **14** |
| 2 | **5** | **42** | **462** |
| 3 | **14** | **462** | **24024** |
| 4 | **42** | **6006** | **1662804** |
| 5 | **132** | **87516** | *140229804* |
| 6 | **429** | **1385670** | *13672405890* |
| 7 | **1430** | *23371634* | |
| 8 | **4862** | *414315330* | |
| 9 | **16796** | | |
| 10 | **58786** | | |

The Catalan numbers are very common combinatorial numbers and show up all over the place. The main proposal for proving this conjecture would be to find a bijection between the process for ordering the bundles and something else counted by the Catalan numbers[Bru10].

We do not have a solution yet for counting the number of bundle rankings when given a number os spaces instead of a set of bundles to rank.

## Deducing income and substitution effects without using indifference

Up till now we have not utilized any assumptions involving budget constraints, everything could have been considered in a world of pure barter. In this section we will work with budget constraint assumptions in order to show that using the Slutsky method it is possible to deduce income and substitution effects without indifference. Gary Becker has been doing this for quite some time,

but apparently never realized it was useful in the debate between Austrian economists and the mainstream [Bec71].

Assume that some person has 6 units of money that they are value less valuable to him then just about anything else, they are burning a whole in his pocket and he wants to get rid of them. Some good $A$ is priced at 1 unit of money for every 1 unit of the good, and some good $B$ is priced at 3 units of money for every 1 unit of the good. In this situation the possible bundles that our person will choose between are the bundles $(Q_B = 0, Q_A = 6), (Q_B = 1, Q_A = 3)$ and $(Q_B = 2, Q_A = 0)$. Notice that none of these are necessarily preferred to the others, so we can stipulate that that the person prefers $(Q_B = 2, Q_A = 0)$ to the other two.

Now assume the price of good $B$ falls to 2 units of money a unit. In this case the three possible choices are $(Q_B = 0, Q_A = 6), (Q_B = 1, Q_A = 4), (Q_B = 2, Q_A = 2)$ and $(Q_B = 3, Q_A = 0)$. In this case we can stipulate that the person prefers the bundle $(Q_B = 1, Q_A = 4)$. This violates no rules whatsoever in the mainstream system $(Q_B = 1, Q_A = 4)$ could be preferred to all these bundles as well as all the bundles at the original price.

The way the Hicks decomposition would tell you to separate income from substitution effect would be to instruct you to find a bundle that you would be indifferent to if compared the $(Q_B = 2, Q_A = 0)$ bundle you selected at the original price, meaning it would not be possible under our modified mainstream system which does not have indifference.

However, we can still use the Slutsky decomposition method. In this method we ask what amount of additional money would we have to possess at the original price to allow you to just afford the new bundle. In order to afford the bundle $(Q_B = 1, Q_A = 4)$ at the price 1 unit of money for every 1 unit of good $A$, good $B$ is priced at 3 units of money for every 1 unit of the good we would need 1

additional unit of money to make 7 units of money.

At these prices and with this budget constraint the possible bundles to purchase are Unless we had started by picking a particular ranking we would at this point be unsure whether with this amount of money at this price are $(Q_B = 0, Q_A = 7), (Q_B = 1, Q_A = 4)$ and $(Q_B = 2, Q_A = 1)$ all of which are preferred to the possible bundles when 1 unit less of money is had. However, it is unclear from the information given in which order they are ranked.

This means that given simply information about preference revealed in action it is impossible to know the magnitude of the income and substitution effect. This is unsurprising, however it does not mean that those things are not implied by action. They certainly seem to exist intuitively. However a problem emerges at this point. Given the possible choices from the increase in income the income effect is either 0 or $-2$. However, if the income effect is 0 then the substitution effect would be $-1$ which seems impossible. In the other case a different substitution effect of 1 is yielded. The question at this point becomes whether the entire class of preference rankings yeilding a negative substitution effect are ruled out by the Austrian ranking, however that question is beyond the scope of this paper.

## Relations Between the Two Types of Rankings

One possible usage of this information would be to find a relationship between the List of Austrian ranknгs and Mainstream rankings without indifference. The number of Austrian rankings are easy to count. When you are going to order the bundles as before for all bundles $(x, y)$ such that $(x \leq X)$ for some $X$ and $(y \leq Y)$ for some $Y$, then the corresponding Austrian bundles are going to be the ones that rank the first $x$ units of good $A$ and the first $y$ units of good $B$. This is counted by $\binom{x+y}{x}$. Similarly, for placing two goods in any way in

$x$ spaces for the Austrian model is $\sum_{i=0}^{x} \binom{x}{i}$ or $2^x$

While we have yet to generalize a relationship between the Austrian and Mainstream rankings, a basic example demonstrates that some mainstream rankings are pre-empted and not generated by the Austrian rankings, and some Austrian rankings generate more then one mainstream ranking.

suppose we are dealing with a world where the relevant units for a given action are 2 units of some good $A$ and 1 unit of some good $B$. Why this is the case is unimportant, we could imagine that the grocery store is out of everything our actor is interested in except for two units of $A$ and one unit of $B$. There are three possible Austrian style rankings the person could have

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $2nd$ | Good $A$ |
| 3 | $1st$ | Good $B$ |

,

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $A$ |
| 2 | $1st$ | Good $B$ |
| 3 | $2nd$ | Good $A$ |

or

| **Ranking** | **Number** | **Good** |
|---|---|---|
| 1 | $1st$ | Good $B$ |
| 2 | $1st$ | Good $A$ |
| 3 | $2nd$ | Good $A$ |

the first one implies that in the world of the mainstream rankings the bundle $Q_A = 1, Q_B = 0$ is ranked higher then the bundle $Q_A = 0, Q_B = 1$ since the first unit of $A$ is preferred to the first unit of $B$. Additionally we know that $Q_A = 2, Q_B = 0$ is ranked more highly than $Q_A = 1, Q_B = 1$ because the second unit of $A$ is preferred to the first unit of $B$. From our rules of more always being preferred to less we can say that the first Austrian ranking above is consistent with

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 2 | 0 |
| 3 | 1 | 1 |
| 4 | 1 | 0 |
| 5 | 0 | 1 |
| 6 | 0 | 0 |

Similarly,the second Austrian ranking above implies that in the world of the mainstream rankings the bundle $Q_A = 1, Q_B = 0$ is ranked higher then the bundle $Q_A = 0, Q_B = 1$ since the first unit of $A$ is preferred to the first unit of $B$. Additionally we know that $Q_A = 1, Q_B = 1$ is ranked more highly than $Q_A = 2, Q_B = 0$ because the first unit of $B$ is preferred to the second unit of $A$. From our rules of more always being preferred to less we can say that the first Austrian ranking above is consistent with

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 1 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 0 |
| 5 | 0 | 1 |
| 6 | 0 | 0 |

Since we know there are 5 possible mainstream rankings of these bundles we should expect either for the last Austrian ranking to account for all three, or to account for one and show that the Austrian rankings map one to one with the mainstream rankings and eliminate a large number of them. What actually happens is surprising. The last Austrian ranking accounts for exatly two Mainstream rankings

The third Austrian ranking above implies that in the world of the mainstream rankings the bundle $Q_A = 0, Q_B = 1$ is ranked higher then the bundle $Q_A = 1, Q_B = 0$ since the first unit of $B$ is preferred to the first unit of $A$. Additionally we know that $Q_A = 1, Q_B = 1$ is ranked more highly than $Q_A = 2, Q_B = 0$ because the first unit of $B$ is preferred to the second unit of $A$. However, we do not know from our rule of more being preferred to less whether or not our actor prefers the bundle $Q_A = 0, Q_B = 1$ to the bundle $Q_A = 2, Q_B = 0$ or vice versa. As a result two possible mainstream rankings result:

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 1 | 1 |
| 3 | 0 | 1 |
| 4 | 2 | 0 |
| 5 | 1 | 0 |
| 6 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 1 | 1 |
| 3 | 2 | 0 |
| 4 | 0 | 1 |
| 5 | 1 | 0 |
| 6 | 0 | 0 |

As mentioned above one of the theoretically possible mainstream rankings is pre-empted by the Austrian rankings. The ranking

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 2 | 0 |
| 3 | 1 | 1 |
| 4 | 0 | 1 |
| 5 | 1 | 0 |
| 6 | 0 | 0 |

fails because it implies the $2nd$ unit of $A$ is preferred to the $1st$ unit of $B$ by saying that the bundle $Q_A = 2, Q_B = 0$ is preffered to the bundle $Q_A = 1, Q_B = 1$. But it also claims that the $1st$ unit of $B$ is preferred to the $1st$ unit of $A$ by saying that the bundle $Q_A = 0, Q_B = 1$ is preferred to the bundle $Q_A = 1, Q_B = 0$.

Since Austrians would say the $1st$ unit of $A$ is always preffered to the $2nd$ unit of $A$, the above preference violate transitivity.

From our rules of more always being preferred to less we can say that the first Austrian ranking above is consistent with

## Creating Utility Functions

Another potential problem is coming up with utility functions that do not yield any indifference. One obvious soloution would be

$$U(x,y) = y(\prod_{i=1}^{x} p_i) \text{ where } p_1 \text{ is the } i\text{th prime}$$

Another possible solution involves introducing a second equation that you only look to to break ties that result from the first equation. The very basic utility functions we are working with are taken from Varian's Microeconomics[Var06]. It is easy to show that if you restrict the values of $x$ and $y$ such that $x > 0$ and $y > 0$ then for any equation of the form $U_1(x,y) = x^n y^m$ a second equation of the form $U_2(x,y) = x^{n+r} y^m$ where $r \neq 0$ will never yield a tie for the same values of $x$ and $y$. If we start by assuming the opposite

$$U_1(x_1,y_1) = U_2(x_2,y_2)$$
$$x_1^n y_1^m = x_2^n y_2^m$$
and
$$U_1(x_1,y_1) = U_2(x_2,y_2)$$
$$x_1^{n+r} y_1^m = x_2^{n+r} y_2^m$$

where either $x_1 \neq x_2$ or $y_1 \neq y_2$. we end up with

$$\frac{x_1^{n+r} y_1^m}{x_1^n y_1^m} = \frac{x_2^{n+r} y_2^m}{x_2^n y_2^m}$$
$$x_1^r = x_2^r$$

$$x_1 = x_2$$

so

$$y_1 = y_2$$

which is a contradiction.

A similar result exists for utility functions of the form $U_1(x, y) = nx + mv$ and $U_1(x, y) = anx + my$. $x$ need not be greater than 0 in this case though, just not equal to 0.

## Conclusion

Many interesting things seem to emerge from this mathematical treatment of the foundational parts of mainstream and Austrain economics. This work may be a good foundation for Austrian dialogue with mainstream in the future, both as a means to critique problems and to ameliorate disputes. However, undoubtedly it is far from complete and barely even started.

## Appendix: Compendium of Possible Rankings for Low Values of X and Y and for Low Numbers of Spaces to be filled

### Possible Rankings when $X, Y \leq 1$

| $U_{xy}$ | $Q_x$ | $Q_y$ | $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 2 | 1 | 0 |
| 3 | 1 | 0 | 3 | 0 | 1 |
| 4 | 0 | 0 | 4 | 0 | 0 |

**Possible Rankings when $X \leq 1$ and $Y \leq 2$**

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 1 | 1 |
| 3 | 1 | 0 |
| 4 | 0 | 2 |
| 5 | 0 | 1 |
| 6 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 1 | 1 |
| 3 | 0 | 2 |
| 4 | 1 | 0 |
| 5 | 0 | 1 |
| 6 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 1 | 1 |
| 3 | 0 | 2 |
| 4 | 0 | 1 |
| 5 | 1 | 0 |
| 6 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 0 | 2 |
| 3 | 1 | 1 |
| 4 | 1 | 0 |
| 5 | 0 | 1 |
| 6 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 0 | 2 |
| 3 | 1 | 1 |
| 4 | 0 | 1 |
| 5 | 1 | 0 |
| 6 | 0 | 0 |

**Possible Rankings when $X \leq 2$ and $Y \leq 2$**

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 1 | 0 |
| 7 | 0 | 2 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 2 | 0 |
| 5 | 1 | 1 |
| 6 | 1 | 0 |
| 7 | 0 | 2 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 2 | 0 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 2 | 0 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 0 | 2 |
| 5 | 2 | 0 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 1 | 1 |
| 6 | 1 | 0 |
| 7 | 0 | 2 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 0 | 2 |
| 6 | 0 | 1 |
| 7 | 2 | 0 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 0 | 1 |
| 7 | 2 | 0 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 0 | 2 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 2 | 0 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 0 | 2 |
| 6 | 2 | 0 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 2 | 0 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 3 | 2 | 0 |
| 4 | 1 | 2 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 1 | 0 |
| 7 | 0 | 2 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 2 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 2 | 0 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 1 | 1 |
| 5 | 0 | 2 |
| 6 | 2 | 0 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 2 | 0 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 0 | 2 |
| 5 | 2 | 0 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 0 | 2 |
| 4 | 2 | 1 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 0 | 2 |
| 4 | 2 | 1 |
| 5 | 2 | 0 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 1 | 1 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 2 | 0 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 1 | 0 |
| 8 | 0 | 1 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 0 | 2 |
| 4 | 2 | 1 |
| 5 | 1 | 1 |
| 6 | 0 | 1 |
| 7 | 2 | 0 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 1 | 1 |
| 5 | 2 | 0 |
| 6 | 0 | 2 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 2 | 0 |
| 5 | 1 | 1 |
| 6 | 0 | 2 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 2 | 0 |
| 5 | 0 | 2 |
| 6 | 1 | 1 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 0 | 2 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 2 | 1 |
| 4 | 0 | 2 |
| 5 | 2 | 0 |
| 6 | 1 | 1 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 0 | 2 |
| 4 | 2 | 1 |
| 5 | 1 | 1 |
| 6 | 2 | 0 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 0 | 2 |
| 4 | 2 | 1 |
| 5 | 2 | 0 |
| 6 | 1 | 1 |
| 7 | 0 | 1 |
| 8 | 1 | 0 |
| 9 | 0 | 0 |

## Possible Rankings Over 3 Spaces

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 1 | 0 |
| 3 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 0 | 1 |
| 3 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 2 |
| 2 | 0 | 1 |
| 3 | 0 | 0 |

## Possible Rankings Over 4 Spaces

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 0 | 1 |
| 3 | 1 | 0 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 0 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 0 |
| 2 | 0 | 1 |
| 3 | 1 | 0 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 2 |
| 2 | 0 | 1 |
| 3 | 1 | 0 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 2 |
| 2 | 1 | 0 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 0 | 2 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 2 | 0 |
| 3 | 1 | 0 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 3 | 0 |
| 2 | 2 | 0 |
| 3 | 1 | 0 |
| 4 | 0 | 0 |

| $U_{xy}$ | $Q_x$ | $Q_y$ |
|---|---|---|
| 1 | 0 | 3 |
| 2 | 0 | 2 |
| 3 | 0 | 1 |
| 4 | 0 | 0 |

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